The input impedance of an alp horn including an Alexander mouthpiece

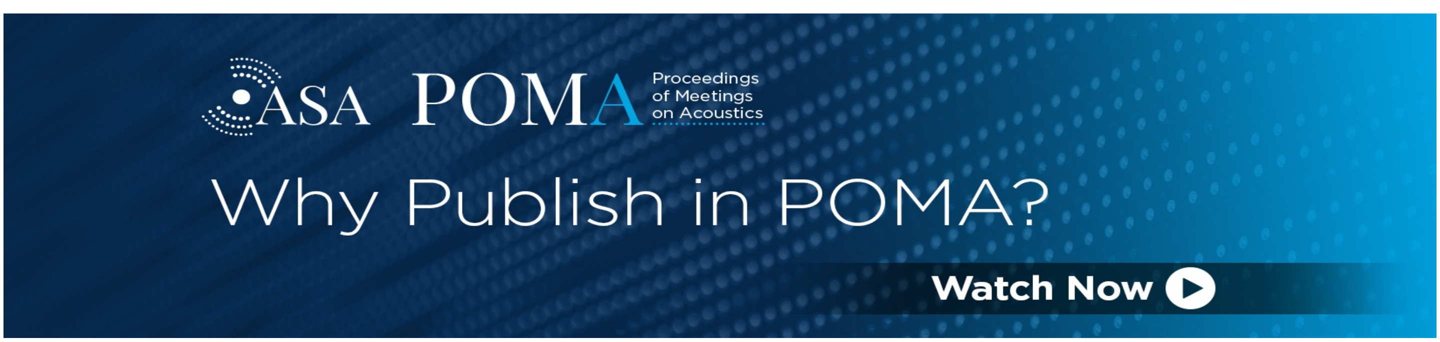
Frederick J. Young

Citation: Proc. Mtgs. Acoust. **6**, 035002 (2009); doi: 10.1121/1.3187533
View online: https://doi.org/10.1121/1.3187533
View Table of Contents: https://asa.scitation.org/toc/pma/6/1
Published by the Acoustical Society of America

ARTICLES YOU MAY BE INTERESTED IN

Objective evaluation of musical instrument quality: A grand challenge in musical acoustics.
Proceedings of Meetings on Acoustics **19**, 032003 (2013); https://doi.org/10.1121/1.4801037

Traditional brass instruments of the world.
The Journal of the Acoustical Society of America **128**, 2282 (2010); https://doi.org/10.1121/1.3507991



Proceedings of Meetings on Acoustics

Volume 6, 2009 http://asa.aip.org

157th Meeting
Acoustical Society of America
Portland, Oregon
18 - 22 May 2009
Session 2pMUa: Musical Acoustics

2pMUa7. The input impedance of an alp horn including an Alexander mouthpiece

Frederick J. Young*

***Corresponding author's address: Communications and Arts, University of Pittsburgh at Bradford, PA, 800 Minard Run Road, Bradford, PA 16701, youngfj@youngbros.com**

The Alp horns built and sold by the late Joseph Littletown of Hammondsport, NY are analyzed. His dimensions of bore as a function of position are used in the solution of the Webster equation for impedance solved numerically earlier for the flügelhorn and the trombone[3]. In addition, the steady flow through the instrument is calculated by Laplace equation. The musical notation for the frequencies of the impedance peaks found by the solutions to the Webster equation are given below. Here the subscripts refer to the harmonic number. The fundamental, rarely used, is 12 cents sharp from Eb at 51.9 Hertz. The intonation errors for C1, C2, G3, C4, E5, G6, B7b, C8, D9, E10, F11, G12, A13, B14b B15, C16, C17# and D18 are 269, 30, 63, -39, -41 ,-25, -13, -7, -11, -1, 2, 11, 11, 15, 15, 20, 24 and 28 cents.

Published by the Acoustical Society of America through the American Institute of Physics

©2009 Acoustical Society of America [DOI: 10.1121/1.3187533]
Received 31 May 2009; published 7 Jul 2009

1. INTRODUCTION

Although Alphorns have been in use for centuries, their bore as a function of station along the length of the instrument is not well known and in some cases is treated as proprietary. The analyses presented here would not have been possible without the cooperation of the late Joseph Littleton, an Alphorn maker formerly of Hammondsport, NY. The dimensions of his Alphorns are used along with the dimensions of horn mouthpieces that are well known. The inner dimensions are given in Figure 1 where the scale for the bore radius is greatly magnified compared to the scale for the distance along the Alphorn. The mouthpiece is situated at the lower right corner of the figure. In our calculations no allowance is made for the bend that is built near the end of the bell and refections from the floor are neglected. Observations of carbon fiber Alphorns that are light enough to hold like a trombone indicate that the bend in the bell and the effect of the floor makes no difference in the sound.

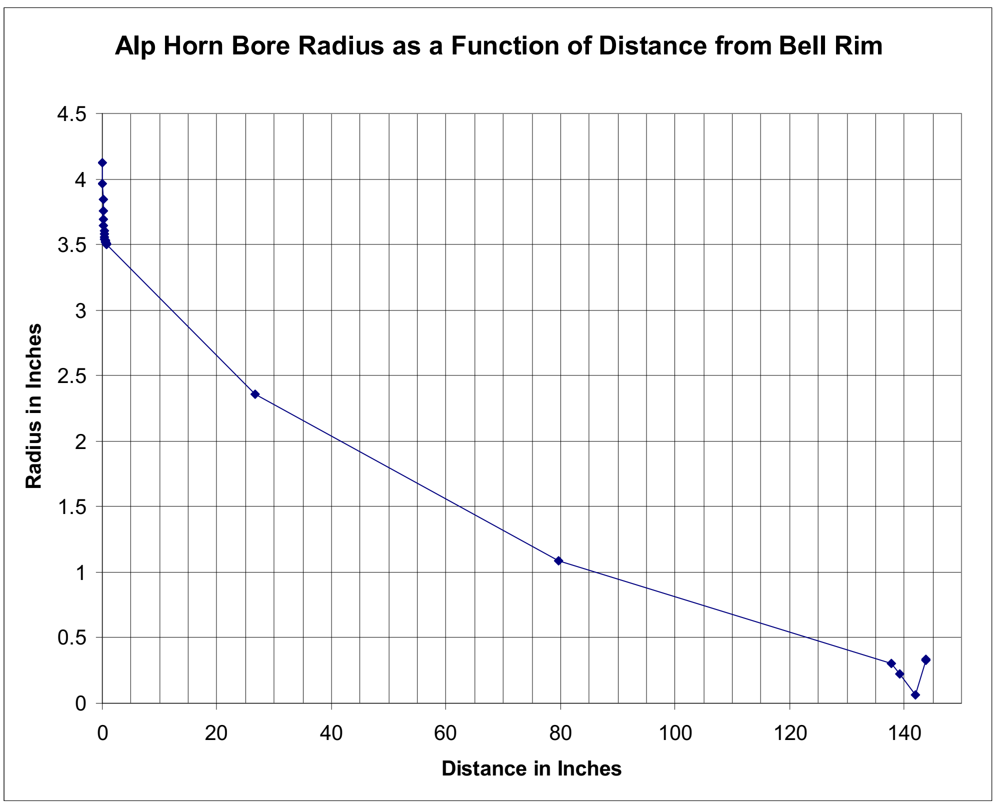


FIGURE 1. Alphorn internal dimensions

STEADY FLOW IN THE ALPHORN

The Alphorn is placed in the center of a cylindrical chamber of 6.24 m wide and 6.65 m tall. This chamber is roughly the size of a typical music practise room. The mouthpiece has a stream of air flowing through its middle as if coming from lips

1

that are 1 cm wide. The physical length of the Alphorn and mouthpiece is 365 cm. The Alphorn body is about 1/2 cm in thickness. A potential flow problem is solved for this geometry to obtain the effective length of the instrument. The appropriate equation for the flow is

$$\nabla \cdot \nabla \varphi = 0 \tag{1.1}$$

where φ is the potential. The temperature is assumed to be 24.2 °C yielding the sonic velocity in air of c_s = 33245 cm/sec and an atmospheric pressure of 119.53 kPa. Let $P = P_0\varphi$ and the velocity vectors are given by

$$\mathbf{V} = -\boldsymbol{\nabla\varphi/40} \tag{1.2}$$

At the mouthpiece z = 0 and for $0 \leq r \leq 0.5$ the boundary condition is $\varphi = const$ and for $0.5 \leq r \leq 312$, $\partial\varphi/\partial n = 0$. This is the condition for all boundaries except the top where z = 624 and on that surface $\varphi = 0$. Eq. 1.1 is solved by the use of finite elements.

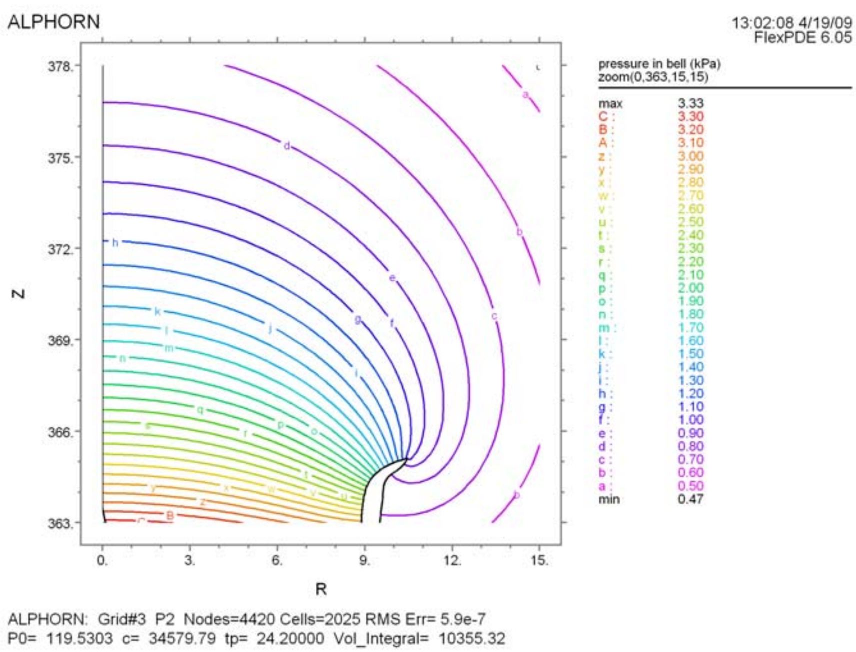


FIGURE 2. Pressure in the vicinity of the bell

Figure 2 shows the pressure distribution in the vicinity of the end of the bell. This figure indicates the effective length of the Alphorn may be about seven or eight cm longer than the dimensions of Figure 1. The velocity in the same region is exhibited in Figure 3 and indicates velocity along the axis of the bell diminishes faster than does the pressure. However, at larger values of radius the velocities tend to increase. Because of the two opposite effects the effective length of the bell is kept at 365 cm for the solution to the Webster equation.

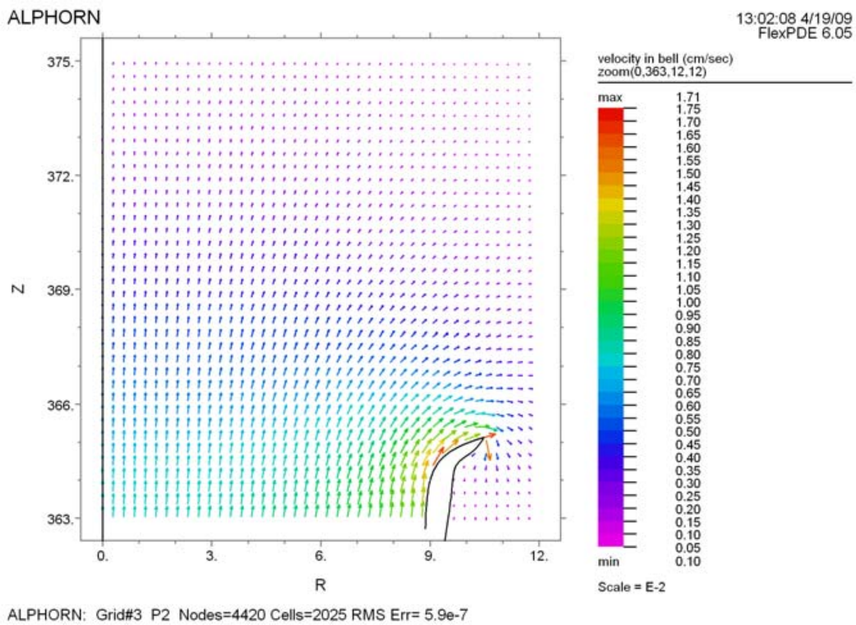


Figure 3. Velocity vectors in the vicinity of the bell

2. One dimensional solution

The impedance of the Alphorn seen looking into the mouthpiece will be found by solving numerically the complex Riccati differential equation that accounts for the variation of bore as a function of position along the instrument [1]. A circular massless piston in an infinite baffle is loading the bell of the Alphorn. Its impedance is complex because it accounts for losses beyond the bell. In that case the impedance seen at any station along the instruments is also complex and denoted as **Z(x).** Here x is the distance from the bell toward the mouthpiece in cm, r(x) is the inside radius of the bore in cm, ω is the angular excitation frequency ρ is the density of the air in grams per cm^3, c is the velocity of sound in centimeters per second, and $j = (-1)^{1/2}$. Then the impedance is given by

$$d\mathbf{Z}(x)/dx = j\omega\{\rho/\pi r^2(x) - (\pi/\rho)[r(x)\mathbf{Z}(x)/c]^2\} \tag{2.1}$$

The termination $\mathbf{Z}_0$ is given by Olson [2] as

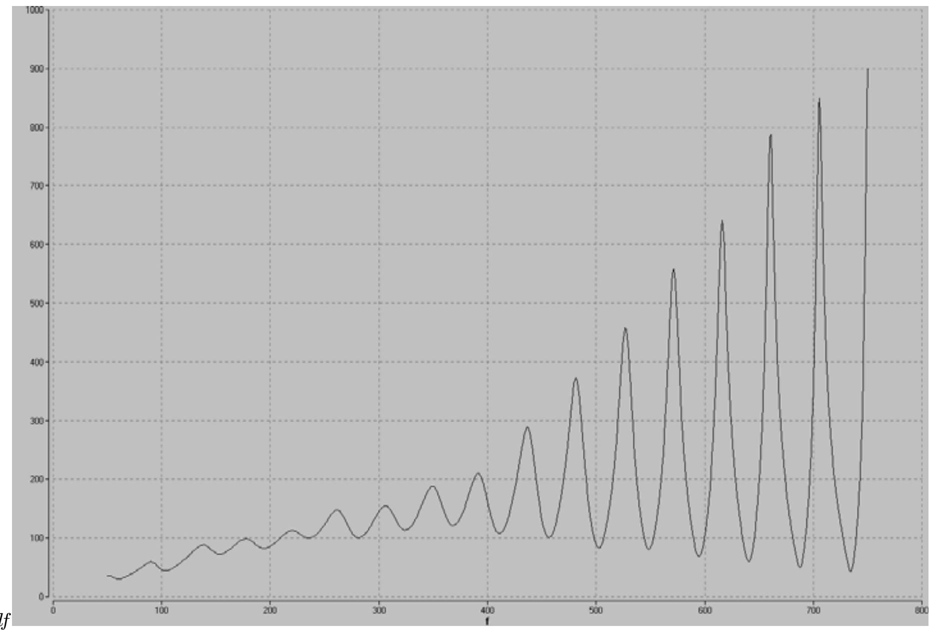
$$\mathbf{Z}_0 = (\rho c/\pi r_0^2)[1 - J_1(2kr_0)/kr_0] + [j\omega\rho/(2\pi r_0^4 k^3)K_1(2kr_0)] \tag{2.2}$$

where J_1 and K_1 are Bessel and Struve functions of the first kind and zeroth order. These are the equations used by Young [3] and derived [4] to obtain the natural frequencies of the flügelhorn and the trombone without mouthpieces. Equation 2.1 is solved by Berkeley Madonna software using the 4th order Runge-Kutta method. The program listing is given in the Appendix. The program is used first to find the approximate values of the peaks of the curve of the | $\mathbf{Z}(0)$ | versus frequency as shown in Figure 4. The software is arranged so that the vicinity of the peaks can be

4

expanded to accurately find the frequency at the maximum value of the magnitude of the input impedance. Figure 5 exhibits the values of said peaks given in cents deviation from the musical notes listed.

graph



4.pdf

FIGURE 4. The magnitude of the input impedance vs frequency

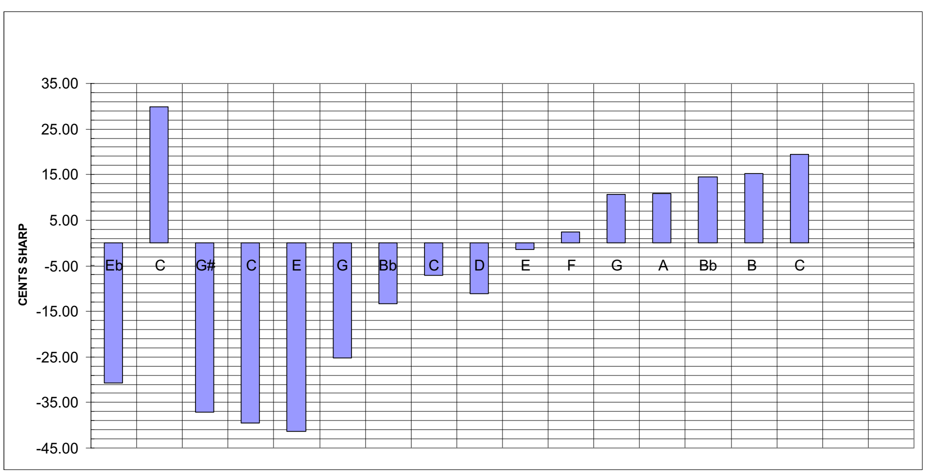


FIGURE 5. Intonation of the Alphorn with Alexander mouthpiece

3. Verification by Finite Element Solution

Using finite elements the eigenvalues of the room and the alphorn can be calculated. The equation for the potential is given by

$$\nabla^2\varphi + \lambda^2\varphi = 0$$

where λ the eigenvalue and the natural frequency of the alphorn is $f = \lambda c/(2\pi)$. Here c is the sonic speed. The pressure in the vicinity of the bell at the first alphorn resonance is given in Figure 6. The resonant frequency is 52 Hz. In Figure 7 the velocity vectors in the vicinity of the bell are shown. The pressure and velocity vectors in the mouthpiece are exhibited in Figures 8 and 9. The resonant frequency of the whole space is about 13 Hz which leads to many closely spaced resonances not related to the alphorn.

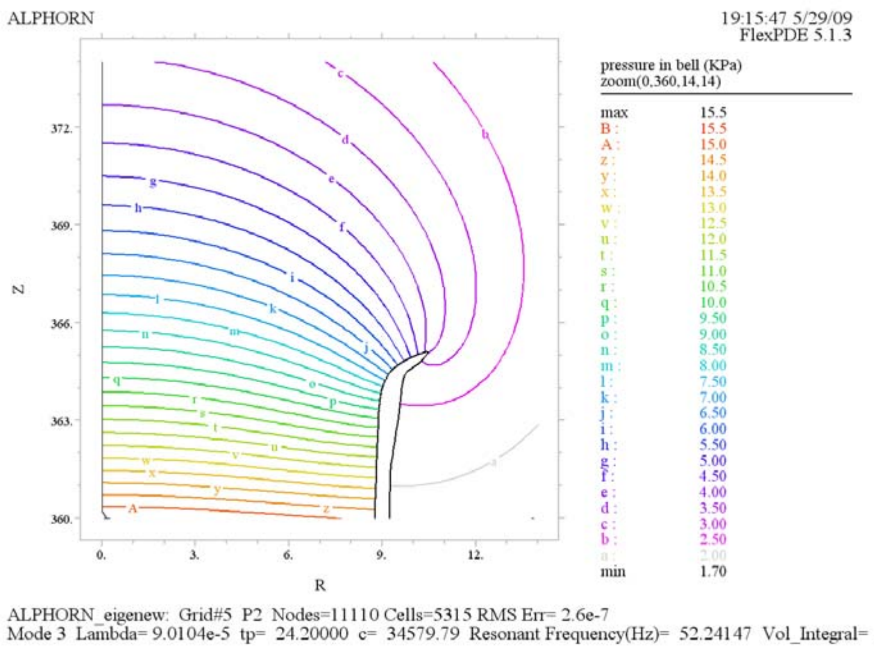


Figure 6. Pressure in the vicinity of the bell at the first alphorn resonance

Figure 10 shows the pressure distribution along the axis of the alphorn. It peaks about 80 cm from the mouthpiece.

Figures 8 and 9 show the region ($r \lesssim 0.48$) where the lips are vibrating. The resonant frequency calculated by the finite element solution of the Helmholtz equation is 52 Hertz. The one dimensional solution yields 51.1 Hertz

4. Discussion and Conclusions

The fundamental of the waldhorn in F is a low F at 43.56 Hertz and is easily produced by most low hornists. It might be expected that for an Alphorn in F the fundamental would be the same. However, the calculations yield a fundamental

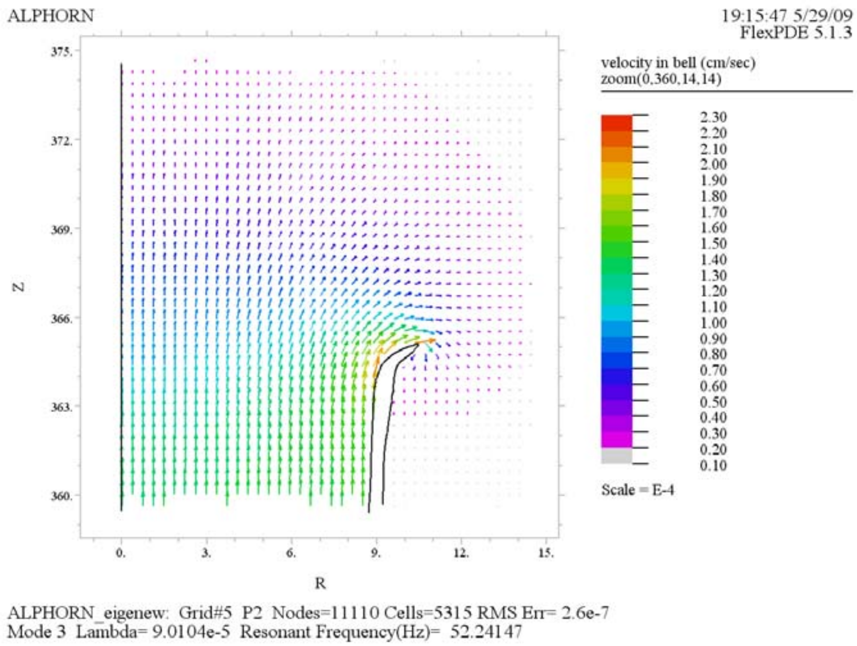


Figure 7. Velocity vectors in the vicinity of the bell

at 51.1 or 52 Hertz. After the calculations were made Joe Littleton said he could not play that low and the fundamental is rarely used. A low hornist tried the Alphorn and produced an A^b(based on A_{440}) that was noticeably flat verifying the calculations. It would seem physically that the great length of cylindrical tubing in the waldhorn lowers the fundamental to 43.56 Hertz. The other errors in intonation were also verified experimentally. It is noteworthy that the Littleton Alphorn is well in tune in the mid to upper range and probably can be lipped in tune elsewhere.

5. Appendix 1

Here is a listing of the script used in the Berkeley Madonna program to solve the complex Riccati differential equation set forth as Equation 2.1. It is split into first order differential equations in the real,U and imaginary, V parts of the complex impedance. The most difficult part of the script involves the calculation of the Bessel and Struve functions which is straightforward but messy. Below $PI = \pi$. $U' = dU/dx$ and $V' = dv/dx$.

5.1. **Berkeley Madonna Script.** RENAME TIME = X RENAME STARTTIME = X0 RENAME STOPTIME = Xf DT = 1 X0 = 0 Xf = 143.7*2.54

init U = ZLr init V = ZLi

X1=0.75*2.54 X2=26.75*2.54 X3=79.75*2.54 X4=139.25*2.54 X5=142.75*2.54

R1=3.5*2.54 R2=2.356*2.54 R3=1.084*2.54 R4=0.2*2.54 R5=0.1045*2.54 Rf=0.3464567*2.54

rho=0.0013876 c=34337 alf=10.4775 f = 90 om = 2*PI*f k = om/c delta=2*k*alf rcc=rho*c^2

q=delta/3 p = 1/q

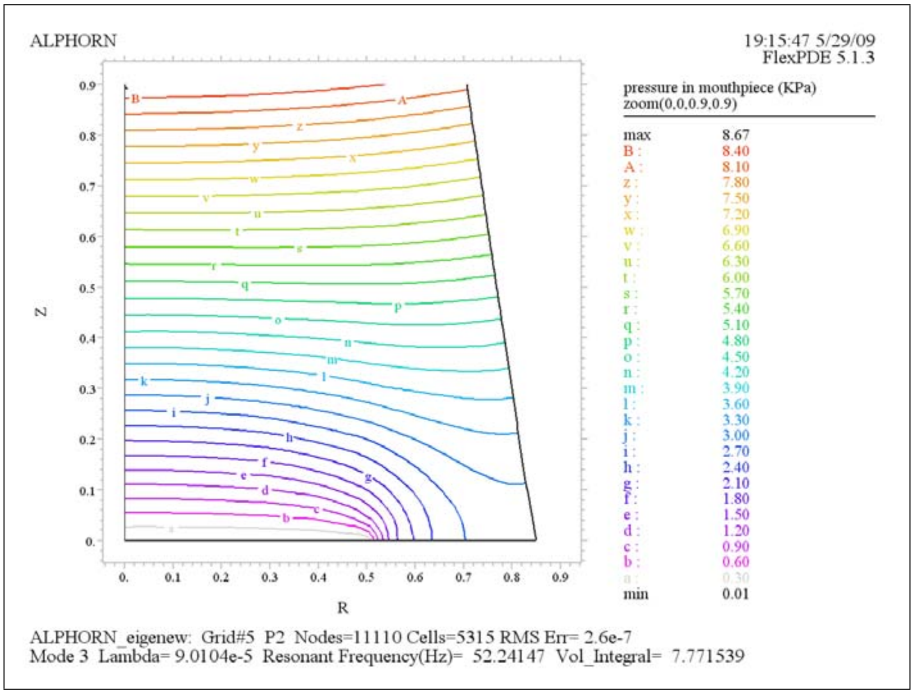


FIGURE 8. Pressure in the vicinity of the mouthpiece

s1 = 3*q*(-0.56249985*q^2 + 0.21093573*q^4 - 0.03954289*q^6 + 0.00443319*q^8 - 0.00031761*q^10 + 0.00001109*q^12)

th1 = 3*q - 2.35619449 + 0.12499612*p + 0.0000565*x^2 - 0.00637879*p^3 +0.00074348*p^4 + 0.00079824*p^5 - 0.00029166*p^6

s2 = (1/(delta^0.5))*cos(th1)*(0.79788456 + 0.00000156*p + 0.01659667*p^2 + 0.00017105*p^3 - 0.00249511*p^4 + 0.00113653*p^5 - 0.00020033*p^6)

bessj= if delta<=3 then s1 else s2

ZLr=(rho*c/(PI*alf*alf))*(1-bessj/(k*alf))

ZLi=(rho*c/(PI*alf*alf))*((2/PI)*(delta^3)*(1/3-(1/45)*delta^2 + (1/1575)*delta^4))

r= if x<=X1 then 2.54*(3.5+0.01489935234*sinh(5.906297*(0.75-x/2.54))) else if x>X1 AND x<=X2 then R1+0.044*(X1-x) else if x>X2 AND x<=X3 then R2+ 0.024*(X2-x) else if x>X3 AND x<=X4

then R3+0.014857*(X3-x) else if x>X4 AND x<=X5 then R4+0.0272857*(X4-x) else if x>X5 AND x<=Xf then R5-0.23014567*(X5-x) else Rf

pirr=PI*r^2

U' = 2*pirr*om*U*V/rcc

V' = -om*((pirr/rcc)*(U*U-V*V)-rho/pirr)

magsq = U^2+V^2

mag=magsq^0.5

mp = (U*U'+V*V')/mag

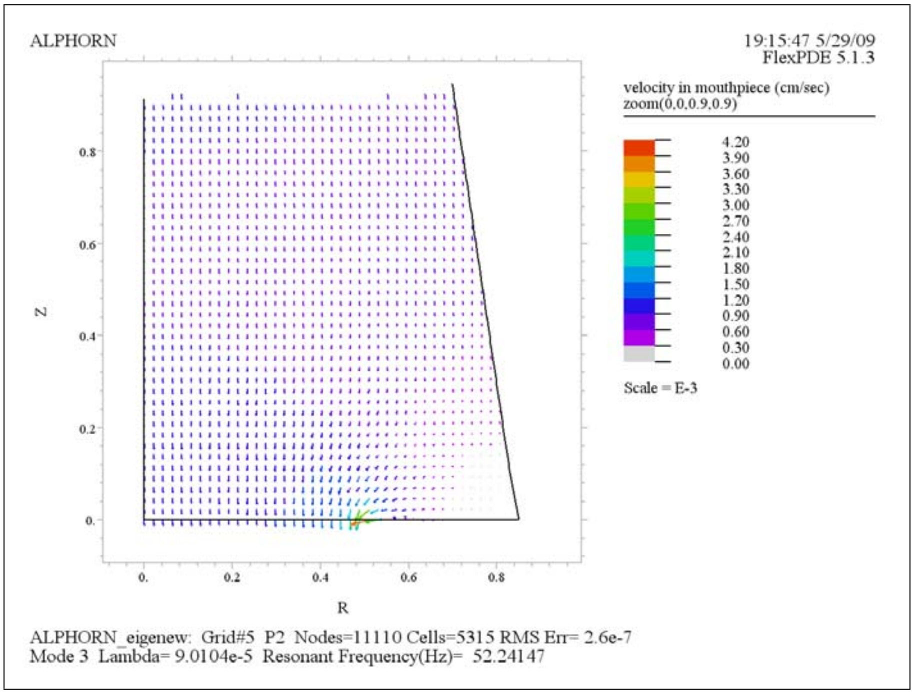


FIGURE 9. Velocity vectors in the mouthpiece

6. APPENDIX 2

Here is the script for programming the calculation of the eigenvalues of the alphorn. The finite element software used is FlexPDE created by PDE Solutions and available at www.pdesolutions.com. Student and trial software is free of charge.

6.1. **FlexPDE script.** Text enclosed in {} brackets are comments ignored by the program and any text following the glif ! is also a comment.

{The outer region is ro = 312 by zo = 665 cm. This is as large as most practise rooms. The boundary at the top of

the Alphorn is obtained from the printed output of the debug statement in the steady flow program.}

title 'ALPHORN'

Select

contours = 20

errlim = 1e-6

modes = 3

Variables

phi

Coordinates

ycylinder(r,z) !cylindrical coordinates

Definitions

tp=24.2 !air temperature

c=33145*(1+tp/273.6)^0.5 !sonic speed

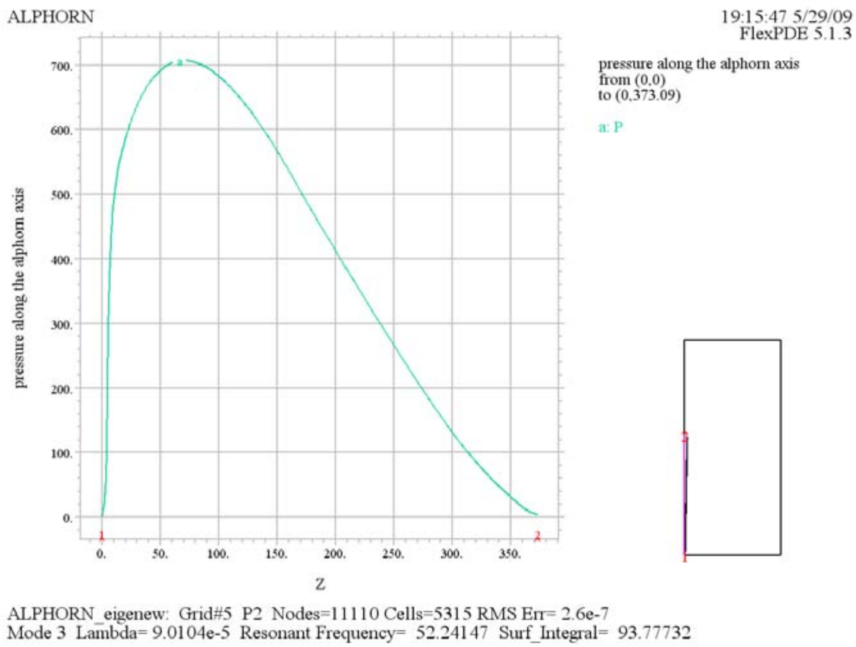


FIGURE 10. Pressure along the alphorn axis

P0 = 101.29*((tp+273.1)/288.08)^5.256 !atmospheric pressure
P = -P0*phi
fe = (0.5*c/Pi)*lambda^0.5 !resonant frequency
vec = -grad(P)/10000 !velocity vectors
Equations
div(grad(phi)) + lambda*phi = 0 !The partial differential equation to be solved subject to Dirichlet and Neumann conditions
Boundaries
Region 1
start (0.85,0) natural(phi)=0 line to (0.8257,0.127) to (0.1565,4.445) to (0.5715,11.43) to (0.7645,15.24) to
(2.7533,162.56) to (5.9842,295) to (8.89,363.22) to (8.9013,363.347) to (8.9136,363.474) to (8.9280,363.601)
to (8.9458,363.728) to (8.9685,363.855) to (8.9980,363.982) to (9.0371,364.109) to (9.0891,364.236)
to (9.1586,364.363) to (9.2516,364.49) to (9.3765,364.617) to (9.5441,364.744) to (9.7691,364.871)
to (10.071,364.998) to (10.477,365.125) spline to(10.4,365) to (10.1,364.7) to(9.8,364.5) to(9.6,363.75)
to(9.5,363) to(9.262,361.474) line to(1.25,11.43) to(1.25,4.445) to(1.25,0.127) to (1.25,0) to (312,0)
to (312,665) value(phi)=0 line to (0,665) natural(phi)=0 line to (0,0) value(phi)=-1 line to (0.5,0) natural(phi)=0
line to close

Plots

contour(P) zoom(0,360,14,14) as 'pressure in bell (KPa)'

report(tp) report(c) report(fe) as 'Resonant Frequency(Hz)'

vector(vec) zoom(0,360,14,14) as 'velocity in bell (cm/sec)'report(fe) as 'Resonant Frequency(Hz)'

contour(P) zoom(0,0,0.9,0.9) as 'pressure in mouthpiece (KPa)' report(fe) as 'Resonant Frequency(Hz)'

vector(vec) zoom(0,0,0.9,0.9) as 'velocity in mouthpiece (cm/sec)' report(fe) as 'Resonant Frequency(Hz)'

elevation(P) from (0,0) to (0,373.09) as 'pressure along the alphorn axis'

report(fe) as 'Resonant Frequency'

End

References

[1] Young, F. J., Sur le calcul des lignes à impédance variable de façon continue (The calculation of continuously variable line impedance), **J. Phys. Radium 19** (1958), 65 - 70

[2] Olson, H. F., **Acoustical Engineering**, Princeton, N. J., (1957), p. 100

[3] Young, F. J., The natural frequencies of musical horns, **Acustica 10**, (1960), 91 - 97

[4] Young, F. J., Posaune akustik (Trombone acoustics), **Die Musik in Geschichte and Gegenwart**, P (1962)