Delayed feedback chaotic oscillator with improved spectral characteristics

Article *in* Electronics Letters · July 2006

DOI: 10.1049/el:20061245 · Source: IEEE Xplore

Delayed feedback chaotic oscillator with improved spectral characteristics

A. Tamaševičius, G. Mykolaitis and S. Bumelienė

A chaotic oscillator, including a nonlinear unit, an amplifier, an RC filter and a delay line, is described. Depending on the gain the circuit exhibits mono- or two-scroll chaotic oscillations. The two-scroll oscillations, in comparison with the mono-scroll oscillations, are characterised by three times higher fundamental frequency.

Introduction: Delayed feedback oscillators are described by a delay differential equation

$$\frac{dx}{dt} = -x(t) + N[x(t-\tau)] \qquad (1)$$

where $x(t)$ is a dynamical variable, $N(\cdot)$ is a nonlinear function and τ is a delay time. The most popular example is the Mackey-Glass (MG) system [1] where $N(\cdot)$ in (1) can be presented by

$$N(x) = \frac{2x}{1+x^{10}} \qquad (2)$$

where $x \equiv x(t-\tau)$ when inserted in (1). An electronic circuit imitating the MG model has been described in [2]. We note that $N(x) \to 0$ at large $|x|$ (practically at $|x| > 2$). Though the $N(x)$ in (2) is an odd-symmetry function the $x(t)$ in (1) oscillates either in the positive ($x > 0$) or in the negative ($x < 0$) region depending on the initial conditions. It does not switch between the two regions, i.e. the system exhibits only simple mono-scroll behaviour. Other nonlinear functions have been also considered for (1), mostly composed of piecewise linear segments. Similarly to the MG nonlinear function (2) they saturate to zero at larger $|x|$. For example, a five-segment function was introduced in [3, 4]:

$$N(x) = \begin{cases} 0, & x \le -\frac{4}{3} \\ -1.5Ax - 2.0, & -\frac{4}{3} < x \le -0.8 \\ Ax, & -0.8 < x \le 0.8 \\ -1.5Ax + 2.0, & 0.8 < x \le \frac{4}{3} \\ 0, & x > \frac{4}{3} \end{cases} \quad (A > 0) \qquad (3)$$

In this Letter we describe a novel delayed feedback oscillator generating not only mono-scroll, but also more complex, two-scroll oscillations with improved spectral characteristics. We consider the following three-segment nonlinear function

$$N(x) = \begin{cases} B(x+1) - A, & x < -1 \\ Ax, & -1 \le x \le 1 \\ B(x-1) + A, & x > 1 \end{cases} \quad (A > 0, B < 0) \qquad (4)$$

and suggest its electronic implementation. In contrast to $N(x)$ in (2) and (3) the $N(x)$ in (4) does not saturate to zero at larger $|x|$.

Circuitry: The oscillator (Fig. 1) has a ring structure and comprises an OA1 based nonlinear unit, an amplifying stage (OA2), a lowpass first-order RC filter, a delay unit DEL described in detail elsewhere [2], and two buffers (OA3, OA4). The slope values $A = ka$ and $B = kb$ in (4) are related to the resistor values as follows:

$$a = \frac{R_2}{R_1 + R_2},\ b = a\left(\frac{R_4}{R_3} + 1\right) - \frac{R_4}{R_3},\ k = \frac{R_6}{R_5} + 1 \qquad (5)$$

The circuit parameters are: $R_1 = R_2 = R_3 = 1$ kΩ, $R_4 = 3$ kΩ ($a = 0.5$, $b = -1.0$), $R_5 = 4.7$ kΩ (trimmer-pot), $R_6 = 10$ kΩ, $R_7 = 190\ \Omega$ (matching resistor), $R = 3.6$ kΩ, $C = 100$ nF ($RC = 360$ µs). The delay time T_{del} was fixed at 3 ms (the dimensionless delay parameter $\tau = T_{del}/RC \simeq 8$). OA1, OA2, OA3 and OA4 are LM741 type opamps; the diodes in the nonlinear unit are general-purpose devices.

Numerical results: Equation (1) with the nonlinear function (4) was integrated numerically. Power spectra (Fig. 2) were obtained from the variable $x(t)$ by means of the fast Fourier transform (FFT) subroutine. At lower values of gain k, which correspond to mono-scroll chaotic oscillations, the fundamental frequency f^* is related to the delay time τ as $f^* \simeq 1/3\tau$ (Fig. 2*a*), whereas at larger values of k, when two-scroll chaotic oscillations are observed, the fundamental frequency is essentially higher: $f^* \simeq 1/\tau$ (Fig. 2*b*). Another interesting feature of the two-scroll mode chaotic oscillations is that the undesired 10–20 dB height peaks emerging at frequencies close to zero, also at the fundamental frequency f^* and its higher harmonics, can be removed (Fig. 2*c*) by adding a constant external force c to (1), similarly to that introduced in [5]. Consequently, the power spectrum becomes flat over a wide frequency band.

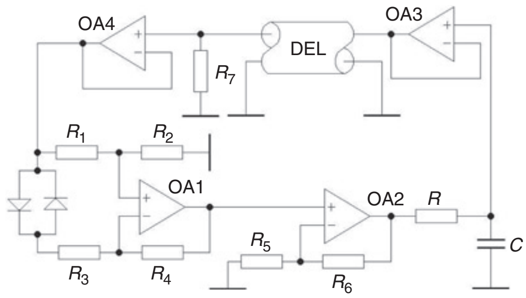


Fig. 1 *Circuit diagram of delayed feedback oscillator*

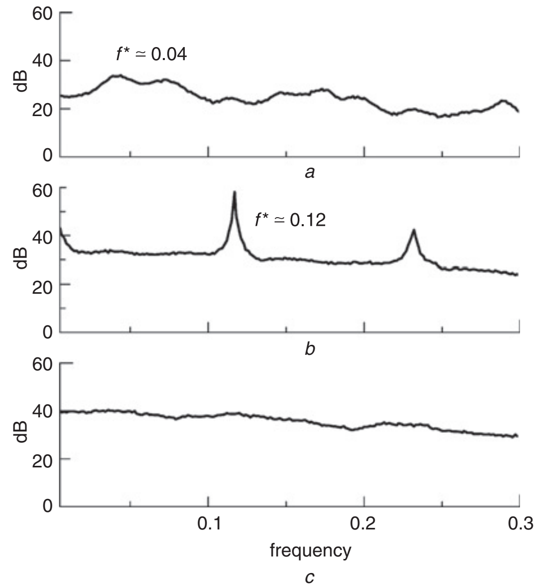


Fig. 2 *Power spectra from (1) with nonlinear function (4) for different values of gain k*

a Mono-scroll oscillations, $k = 3.0$, $c = 0$
b Two-scroll oscillations, $k = 4.0$, $c = 0$
c Two-scroll oscillations, $k = 4.0$, $c = \pm 0.5$
$\tau = 8$, $a = 0.5$, $b = -1.0$

Experimental results: The DC transfer function of the nonlinear unit is shown in Fig. 3. The dynamic behaviour of the overall system is shown in Fig. 4. Power spectra are shown in Fig. 5.

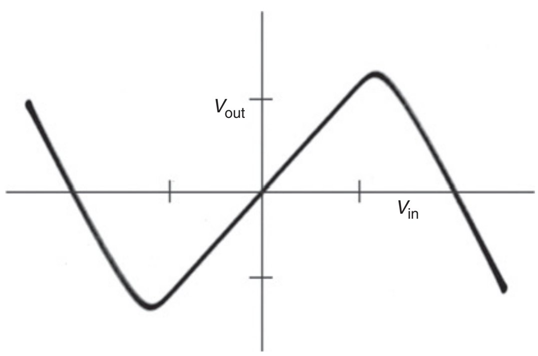
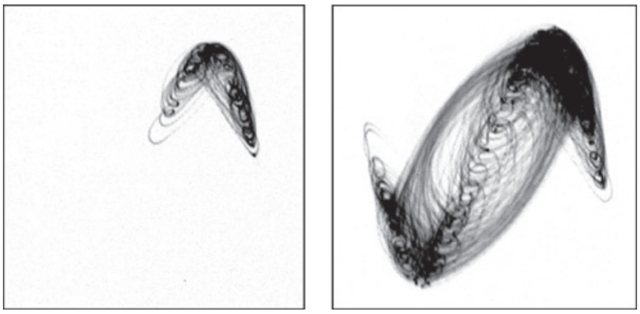


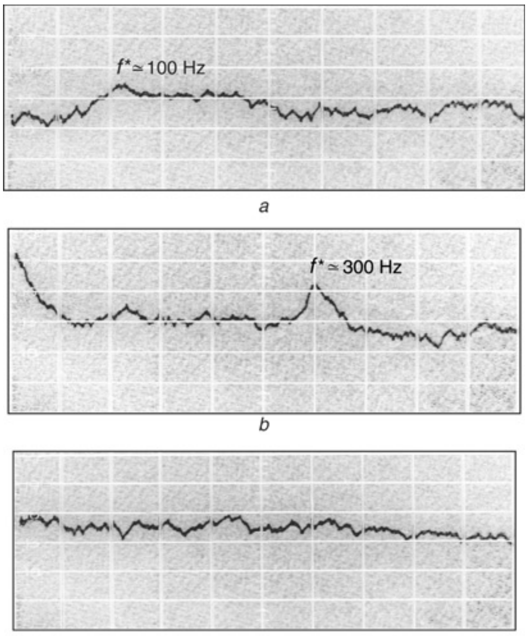
Fig. 3 *Experimental transfer function of nonlinear unit, V_{out} against V_{in}*
Vertical scale 0.5 V/div., horizontal scale 1.0 V/div. $a = 0.5$, $b = -1.0$



a b

Fig. 4 *Experimental phase portraits*

a Mono-scroll oscillations, $k = 3.7$
b Two-scroll oscillations, $k = 4.3$
Vertical $U_C(t)$, horizontal $U_C(t - T_{del})$. $T_{del} = 3$ ms, $a = 0.5$, $b = -1.0$



c

Fig. 5 *Experimental power spectra from circuit in Fig. 1 for different values of gain k*

a Mono-scroll oscillations, $k = 3.7$, no external DC bias
b Two-scroll oscillations, $k = 4.3$, no external DC bias
c Same as in *b*, but with DC bias of 0.2 V applied via resistor R_5
Spectral range 0–500 Hz, horizontal scale 50 Hz/div., vertical scale 10 dB/div., resolution 3 Hz. $T_{del} = 3$ ms, $a = 0.5$, $b = -1.0$

Conclusions: A novel delayed feedback chaotic oscillator is proposed. The delay-line oscillator with a non-saturating nonlinear unit exhibits not only mono-scroll, but also the more complex two-scroll chaotic attractors. The type of oscillations can be controlled by the gain of the amplifier. The fundamental frequency in the two-scroll mode is three times higher than in the mono-scroll mode of chaotic oscillations.

19 April 2006
Electronics Letters online no: 20061245
doi: 10.1049/el:20061245

A. Tamaševičius, G. Mykolaitis and S. Bumelienė (*Plasma Phenomena and Chaos Laboratory, Semiconductor Physics Institute, A. Goštauto 11, LT-01108 Vilnius, Lithuania*)

E-mail: tamasev@pfi.lt

References

1 Marek, M., and Schreiber, I.: 'Chaotic behaviour of deterministic dissipative systems' (Cambridge University Press, Cambridge, 1991)
2 Namajūnas, A., Pyragas, K., and Tamaševičius, A.: 'An electronic analog of the Mackey-Glass system', *Phys. Lett. A*, 1995, **201**, (1), pp. 42–46
3 Lu, H., and He, Z.: 'Chaotic behavior in first-order autonomous continuous-time systems with delay', *IEEE Trans. Circuits Syst. I*, 1996, **43**, (8), pp. 700–702
4 Lu, H., He, Y., and He, Z.: 'A chaos-generator: analyses of complex dynamics of a cell equation in delayed cellular neural networks', *IEEE Trans. Circuits Syst. I*, 1998, **45**, (2), pp. 178–181
5 Thangavel, P., Murali, K., and Lakshmanan, M.: 'Bifurcation and controlling of chaotic delayed cellular neural networks', *Int. J. Bifurcation Chaos Appl. Sci. Eng.*, 1998, **8**, (12), pp. 2481–2492